Int J Theor Phys (2010) 49: 1863–1869
DOI 10.1007/s10773-010-0364-8

Probabilistic Teleportation of an Arbitrary Two-mode N-photon Entangled State in Cavity QED

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Received: 6 February 2010 / Accepted: 15 April 2010 / Published online: 21 May 2010

Abstract We propose a scheme for teleportation of an arbitrary two-mode N-photon entangled states in cavity QED. The scheme is based on the resonant interaction between Λ-type atoms and two-mode cavity fields. In contrast to all the theoretical schemes proposed previously in cavity QED for teleportation of two-mode cavity field states, in the present scheme, the established entanglement for the quantum channel is the type of the multi-dimensional entanglement between the symmetric multi-atom Dicke states and two-mode N-photon states. Therefore, the scheme extends the scope of the theoretical study of the teleportation.

Keywords Probabilistic teleportation · Two-mode N-photon entangled states · Λ-type atom · Two-mode cavity · Cavity QED

Quantum teleportation, which describes the disembodied transmission of information between two separate subsystems [1], is one of the most useful tools for quantum information processing [2]. It attracts much attention since its first suggestion, because not only the theoretical but also the experimental works have been done. Experimental demonstrations of quantum teleportation have been reported in optical systems [3–6], and ion trap [7, 8].

Cavity quantum electrodynamics (QED), provides a good paradigm for studying quantum teleportation [9, 10]. In the context of cavity QED, schemes have been proposed for teleportation of an unknown quantum state for a single qubit [11–17], and two qubits [18–22]. All these schemes are based on two-state systems. In addition, a great deal of attention has also been paid to the problem of three- or multi-state systems. It has been shown that violations of local realism are stronger for two maximally entangled quNits than for qubits and the former is more resistant to noise [23, 24]. Generic Bell inequalities for multipartite N-state ($N > 2$) systems have been studied [25]. It has been pointed out that, for quantum key distribution, the usage of multi-state systems offers advantages such as increased level of tolerance to noise at a given level of security and a higher bit transmission rate compared

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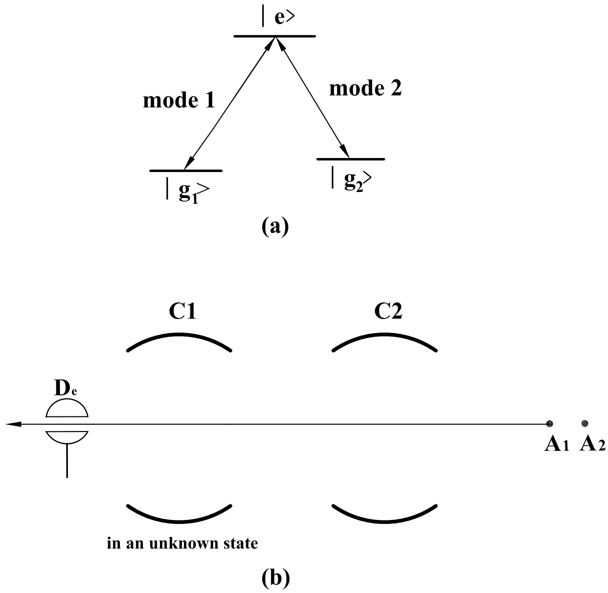

Fig. 1 (**a**) The schematic diagram of a Λ-type atomic level structure interacting with two cavity modes. (**b**) The schematic setup for the teleportation of an arbitrary superposition of two-mode N-photon entangled state

to the qubit QKD protocols [26]. Several schemes that based on cavity QED have been proposed for teleportation of unknown states of three- or multi-state systems, including an arbitrary superposition of three state of an atom [27], an arbitrary superposition of 2^n Fock states in a cavity field [28], and an arbitrary superposition of zero-, one-, and two-photon states in a cavity field [29].

In this paper, we propose a scheme for teleportation of an arbitrary two-mode N-photon entangled state, i.e., $\sum_{i=0}^{N} \alpha_i |N-i, i\rangle_{C_1}$ ($\sum_{i=0}^{N} |\alpha_i|^2 = 1$). In contrast to all the previous schemes [27–29], the entanglement to be established for the quantum channel is a type of the multi-dimensional entanglement between symmetric multi-atom Dicke states and two-mode multi-photon states. The scheme is based on the resonant interaction of a Λ-type three-level atom and a two-mode cavity field. It follows a previous scheme proposed by Zheng [30], which describes that an unknown state of two-state system can be approximately teleported through only a single measurement.

As shown in Fig. 1, the atom has the Lambda configuration with one excited state $|e\rangle$ and two ground states $|g_1\rangle$ and $|g_2\rangle$. We assume the frequencies for the transitions $|e\rangle \leftrightarrow |g_1\rangle$ and $|e\rangle \leftrightarrow |g_2\rangle$ are resonant to the cavity modes 1 and 2, respectively. If the states $|g_1\rangle$ and $|g_2\rangle$ are assumed to be the Zeeman-Rydberg levels thus two cavity modes can be the polarized photon modes. In the interaction picture, the Hamiltonian for the system can be written as [31–33]:

$$H_I = \sum_{i=1,2} g_i \big(a_i |e\rangle\langle g_i| + \text{H.c}\big), \tag{1}$$

where a_i is annihilation operator for the cavity mode i, and g_i is the coupling constant for the corresponding transition. For convenience, we assume two coupling constants to be equal, i.e. $g_1 = g_2 = g$, which can be satisfied by choosing appropriate atomic transitions. We introduce the total excitation number operator $N = \sum_{i=1,2} a_i^+ a_i + |e\rangle\langle e|$. It is apparent that the excitation number operator commutes with the interaction Hamiltonian (1). Therefore, the total excitation number is a conserved quantity during the evolution. Within the

$(m+n+1)$-excitation subspace, the state evolution of the atom-cavity system under the government of the Hamiltonian can be expressed as [31]:

$$\begin{aligned}|e\rangle|m,n\rangle \rightarrow \cos(G_{m,n}\tau)|e\rangle|m,n\rangle - \frac{i\sin(G_{m,n}\tau)}{\sqrt{m+n+2}} \\ \times\left[\sqrt{m+1}|g_1\rangle|m+1,n\rangle + \sqrt{n+1}|f\rangle|m,n+1\rangle\right],\end{aligned} \tag{2}$$

$$\begin{aligned}|g_1\rangle|m+1,n\rangle \rightarrow \frac{1}{m+n+2}\left[(m+1)\cos(G_{m,n}\tau)+n+1\right]|g_1\rangle|m+1,n\rangle \\ + \frac{\sqrt{(m+1)(n+1)}}{m+n+2}\left[\cos(G_{m,n}\tau)-1\right]|g_2\rangle|m,n+1\rangle \\ - i\frac{\sqrt{m+1}}{\sqrt{m+n+2}}\sin(G_{m,n}\tau)|e\rangle|m,n\rangle,\end{aligned} \tag{3}$$

$$\begin{aligned}|g_2\rangle|m,n+1\rangle \rightarrow \frac{1}{m+n+2}\left[(n+1)\cos(G_{m,n}\tau)+m+1\right]|g_2\rangle|m,n+1\rangle \\ + \frac{\sqrt{(m+1)(n+1)}}{m+n+2}\left[\cos(G_{m,n}\tau)-1\right]|g_1\rangle|m+1,n\rangle \\ - i\frac{\sqrt{n+1}}{\sqrt{m+n+2}}\sin(G_{m,n}\tau)|e\rangle|m,n\rangle,\end{aligned} \tag{4}$$

where $G_{m,n} = g\sqrt{m+n+2}$, $|m,n\rangle$ denotes the state with m photons in mode 1 and n photons in mode 2, and τ is the atom-cavity interaction time.

To begin with, let us first consider the teleportation of two-mode two-photon entangled state. The setup for its implementation is shown in Fig. 1(b). Cavity C_1 is initially in the unknown state to be teleported. Cavity C_2 is initialized in the two mode vacuum state $|0,0\rangle$. We will send two atoms A and B through cavities C_2 and C_1 in turn to complete the teleportation process. D denotes the atomic state detector.

Assume cavity C_1 is initially in an arbitrary two-mode two-photon entangled state [14], i.e.,

$$|\psi\rangle_{C_1} = \alpha|2,0\rangle_{C_1} + \beta|1,1\rangle_{C_1} + \gamma|0,2\rangle_{C_1}, \tag{5}$$

with $|\alpha|^2 + |\beta|^2 + |\gamma|^2 = 1$.

Now we want to teleport such a state to cavity C_2. We suppose both atoms are initialized in the excited state $|e\rangle$. We first send atom A through cavity C_2. After an interaction time $\tau_1 = \frac{\pi}{2\sqrt{2}g}$, the system composed of atom A and cavity C_2 evolves to [31]

$$|\psi\rangle_{AC_2} = \frac{-i}{\sqrt{2}}\left(|g_1\rangle_A|1,0\rangle_{C_2} + |g_2\rangle_A|0,1\rangle_{C_2}\right). \tag{6}$$

Then we send atom B through cavity C_2. If the interaction time is chosen to fulfill $\tau_2 = \frac{\pi}{2\sqrt{3}g}$, the system composed of two atoms A, B and cavity C_2 evolves to

$$\begin{aligned}|\psi\rangle_{ABC_2} = -\frac{1}{\sqrt{3}}\Bigg[|g_1\rangle_A|g_1\rangle_B|2,0\rangle_{C_2} + \frac{|g_1\rangle_A|g_2\rangle_B + |g_2\rangle_A|g_1\rangle_B}{\sqrt{2}} \\ \times|1,1\rangle_{C_2} + |g_2\rangle_A|g_2\rangle_B|0,2\rangle_{C_2}\Bigg].\end{aligned} \tag{7}$$

The state $|\psi\rangle_{ABC_2}$ is a type of three-dimensional entangled state between the atoms (if, considered to be unity) and cavity, and will act as the quantum channel for the teleportation. Now the whole system is in

$$|\psi\rangle_{C_1} \otimes |\psi\rangle_{ABC_2}. \tag{8}$$

Then we send atom A through cavity C_1. After an interaction time $\tau_3 = \frac{\pi}{2\sqrt{3}g}$, the whole system evolves to

$$\begin{aligned}|\psi\rangle_{ABC_1C_2} = -\frac{1}{\sqrt{3}}\Big\{&|2,0\rangle_{C_2}|g_1\rangle_B\big(\alpha|\varphi\rangle_{AC_1}^{g_1,2,0} + \beta|\varphi\rangle_{AC_1}^{g_1,1,1} + \gamma|g_1\rangle_A|0,2\rangle_{C_1}\big)\\ &+ \frac{1}{\sqrt{2}}|1,1\rangle_{C_2}\big[|g_2\rangle_B\big(\alpha|\varphi\rangle_{AC_1}^{g_1,2,0} + \beta|\varphi\rangle_{AC_1}^{g_1,1,1} + \gamma|g_1\rangle_A|0,2\rangle_{C_1}\big)\\ &+ |g_1\rangle_B\big(\alpha|g_2\rangle_A|2,0\rangle_{C_1} + \beta|\varphi\rangle_{AC_1}^{g_2,1,1} + \gamma|\varphi\rangle_{AC_1}^{g_2,0,2}\big)\big]\\ &+ |0,2\rangle_{C_2}|g_2\rangle_B\big(\alpha|g_2\rangle_A|2,0\rangle_{C_1} + \beta|\varphi\rangle_{AC_1}^{g_2,1,1} + \gamma|\varphi\rangle_{AC_1}^{g_2,0,2}\big)\Big\},\end{aligned} \tag{9}$$

where

$$|\varphi\rangle_{AC_1}^{g_1,2,0} = \frac{1}{3}|g_1\rangle_A|2,0\rangle_{C_1} - \frac{\sqrt{2}}{3}|g_2\rangle_A|1,1\rangle_{C_1} - i\sqrt{\frac{2}{3}}|e\rangle_A|1,0\rangle_{C_1}, \tag{10}$$

$$|\varphi\rangle_{AC_1}^{g_1,1,1} = \frac{2}{3}|g_1\rangle_A|1,1\rangle_{C_1} - \frac{\sqrt{2}}{3}|g_2\rangle_A|0,2\rangle_{C_1} - \frac{i}{\sqrt{3}}|e\rangle_A|0,1\rangle_{C_1}, \tag{11}$$

$$|\varphi\rangle_{AC_1}^{g_2,1,1} = \frac{2}{3}|g_2\rangle_A|1,1\rangle_{C_1} - \frac{\sqrt{2}}{3}|g_1\rangle_A|2,0\rangle_{C_1} - \frac{i}{\sqrt{3}}|e\rangle_A|1,0\rangle_{C_1}, \tag{12}$$

$$|\varphi\rangle_{AC_1}^{g_2,0,2} = \frac{1}{3}|g_2\rangle_A|0,2\rangle_{C_1} - \frac{\sqrt{2}}{3}|g_1\rangle_A|1,1\rangle_{C_1} - i\sqrt{\frac{2}{3}}|e\rangle_A|0,1\rangle_{C_1}. \tag{13}$$

Here $|\varphi\rangle_{AC_j}^{\nu,m,n}$ denotes the state that evolves from the initial state $|\nu\rangle_A|m,n\rangle_{C_j}$ ($\nu = g_1, g_2$; $m, n = 0, 1, 2$; $j = 1, 2$) after the preset interaction time. Now we perform a measurement on atom A. If this atom is detected in the excited state $|e\rangle_A$, the system composed of atoms B, cavity C_1 and C_2 collapses onto

$$\begin{aligned}|\psi\rangle_{BC_1C_2} = \frac{i}{\sqrt{3}}\Big\{&|2,0\rangle_{C_2}|g_1\rangle_B\big(\sqrt{2}\alpha|1,0\rangle_{C_1} + \beta|0,1\rangle_{C_1}\big)\\ &+ \frac{1}{\sqrt{2}}|1,1\rangle_{C_2}\big[|g_2\rangle_B\big(\sqrt{2}\alpha|1,0\rangle_{C_1} + \beta|0,1\rangle_{C_1}\big)\\ &+ |g_1\rangle_B\big(\beta|1,0\rangle_{C_1} + \sqrt{2}\gamma|0,1\rangle_{C_1}\big)\big]\\ &+ |0,2\rangle_{C_2}|g_2\rangle_B\big(\beta|1,0\rangle_{C_1} + \sqrt{2}\gamma|0,1\rangle_{C_1}\big)\Big\}.\end{aligned} \tag{14}$$

Now we send atom B through cavity C_1. For the interaction time $\tau_4 = \frac{\pi}{2\sqrt{2}g}$, the whole system evolves to

$$
\begin{aligned}
|\psi'\rangle_{BC_1C_2} = \frac{i}{\sqrt{3}}\Big\{ & |2,0\rangle_{C_2}\big(\sqrt{2}\alpha|\varphi\rangle_{BC_1}^{g_1,1,0} + \beta|g_1\rangle_B|0,1\rangle_{C_1}\big) \\
& + \frac{1}{\sqrt{2}}|1,1\rangle_{C_2}\big[\big(\sqrt{2}\alpha|g_2\rangle_B|1,0\rangle_{C_1} + \beta|\varphi\rangle_{BC}^{g_2,0,1}\big) \\
& + \big(\beta|\varphi\rangle_{BC_1}^{g_1,1,0} + \sqrt{2}\gamma|g_1\rangle_B|0,1\rangle_{C_1}\big)\big] \\
& + |0,2\rangle_{C_2}\big(\beta|g_2\rangle_B|1,0\rangle_{C_1} + \sqrt{2}\gamma|\varphi\rangle_{BC}^{g_2,0,1}\big)\Big\}
\end{aligned} \tag{15}
$$

where

$$|\varphi\rangle_{BC_1}^{g_1,1,0} = \frac{1}{2}|g_1\rangle_B|1,0\rangle_{C_1} - \frac{1}{2}|g_2\rangle_B|0,1\rangle_{C_1} - \frac{i}{\sqrt{2}}|e\rangle_B|0,0\rangle_{C_1}, \tag{16}$$

$$|\varphi\rangle_{BC_1}^{g_2,0,1} = \frac{1}{2}|g_2\rangle_B|0,1\rangle_{C_1} - \frac{1}{2}|g_1\rangle_B|1,0\rangle_{C_1} - \frac{i}{\sqrt{2}}|e\rangle_B|0,0\rangle_{C_1}. \tag{17}$$

Then we also perform a measurement on atom B. If this atom is likewise detected in the excited state $|e\rangle_B$, the two cavities are finally in the state

$$|\psi\rangle_{C_1C_2} = \frac{1}{\sqrt{3}}\big(\alpha|2,0\rangle_{C_2} + \beta|1,1\rangle_{C_2} + \gamma|0,2\rangle_{C_2}\big)|0,0\rangle_{C_1}. \tag{18}$$

Therefore, the unknown state of cavity C_1 is faithfully teleported to cavity C_2. The success probability for atomic state detection in each step is $\frac{1}{3}$, thus the total success probability is $\frac{1}{9}$.

The scheme can be generalized for teleportation of arbitrary two-mode N-photon entangled state of the type:

$$|\psi\rangle_{C_1} = \sum_{i=0}^{N}\alpha_i|N-i,\ i\rangle_{C_1}, \tag{19}$$

where $\sum_{i=0}^{N}|\alpha_i|^2 = 1$. In order to teleportate such a state, we need N atoms $A_1, A_2, \ldots,$ and A_N, each one should be initialized in the excited state $|e\rangle_{A_i}$. We send these atoms, in turn, i.e., $A_1, A_2, \ldots,$ and A_N, through the vacuum cavity C_2. Assume the interaction time of each atom with the cavity C_2 is taken to be $t_i = \frac{\pi(i+1)^{-1/2}}{2g}$ $(i = 1, 2, \ldots, N)$, then N atoms and cavity C_2 are prepared in the state

$$|\psi\rangle_{A_1A_2\ldots A_N,C_2} = \sum_{i=0}^{N}|N,i\rangle_{A_1A_2\ldots A_N}|N-i,i\rangle_{C_2}, \tag{20}$$

with

$$|N,i\rangle_{A_1A_2\ldots A_N} = c(i)\big(s_{g_1}^{+}\big)^{i}\big(s_{g_2}^{+}\big)^{N-i}\left(\bigotimes_{i=1}^{N}|e\rangle_{A_i}\right) \tag{21}$$

being the symmetric N-atom Dicke state where i atoms are in the state $|g_1\rangle$ while $N-i$ atoms are in the state $|g_2\rangle$, with $c(i)$ and $s_{g_\nu}^{+}$ $(\nu = g_1,\ g_2)$ being $[N!i!(N-i)!]^{-1/2}$ and $s_\nu^{+} = \sum_{i=1}^{N}|\nu\rangle_{A_i}\langle e|$, respectively [34]. Now the entire system is in

$$|\psi\rangle_{A_1A_2\ldots A_N,C_2}\otimes|\psi\rangle_{C_1}. \tag{22}$$

Then each atom A_i is sent through cavity C_1 in turn, for an interaction time $t_i' = \frac{\pi(N-i+2)^{-1/2}}{2g}$ ($i = 1, 2, \ldots, N$), and later detected, respectively. Suppose that all these atoms are found in the excited state $|e\rangle_{A_i}$, then the state initially stored in cavity C_1 is teleported to cavity C_2, i.e., cavity C_2 is exactly in

$$|\psi\rangle_{C_2} = \sum_{i=0}^{N} \alpha_i |N-i, i\rangle_{C_2}. \tag{23}$$

While cavity C_1 is left in the two-mode vacuum state $|0, 0\rangle_{C_1}$, with the success probability being $\frac{1}{(N+1)^2}$.

In all the above arguments, we have assumed that the dissipation due to the atomic spontaneous emission and photon leakage out of the cavities are negligible. The assumption is valid within the scope of presently available microwave cavity QED technology, which has achieved an ultrahigh finesse Fabry-Perot superconducting resonator [35]. In our scheme, for teleportation of an arbitrary two-mode two-photon entangled state, the total time required for the atom-cavity interaction is $\tau = \sum_{i=1}^{4} \tau_i = \frac{(\sqrt{2}+\sqrt{3})\pi}{\sqrt{6}g}$. By considering the traveling times of the atoms during the teleportation process [36], the required time for the teleportation process might be estimated as $\tau \simeq \frac{40}{g}$, much smaller than the lifetime of the atomic excited state ($> \frac{4000}{g}$) and the cavity field photon ($> \frac{10\,000}{g}$) [35]. The exact control of the interaction time and highly efficient atomic state detection have also been demonstrated to be possible [9, 10].

To summarize, we have proposed a scheme for teleportation of an arbitrary two-mode N-photon entangled state. The scheme is based on the resonant interaction between Λ-type atoms and two-mode cavity fields. In contrast to all the theoretical schemes proposed previously in cavity QED for teleportation of two-mode cavity field states, in the present scheme, the established entanglement for the quantum channel is the type of the multi-dimensional entanglement between the symmetric multi-atom Dicke states and two-mode N-photon states. Therefore, the scheme extends the scope of the theoretical study of the teleportation.

Acknowledgements This work is supported by the Natural Science Foundation of Fujian Province under Grant No. 2008J0217.

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